what light means

prof. dr. nicolas meseth

1. agreement
2. counting information
3. your alphabet

part 1

agreement

what makes light mean something?

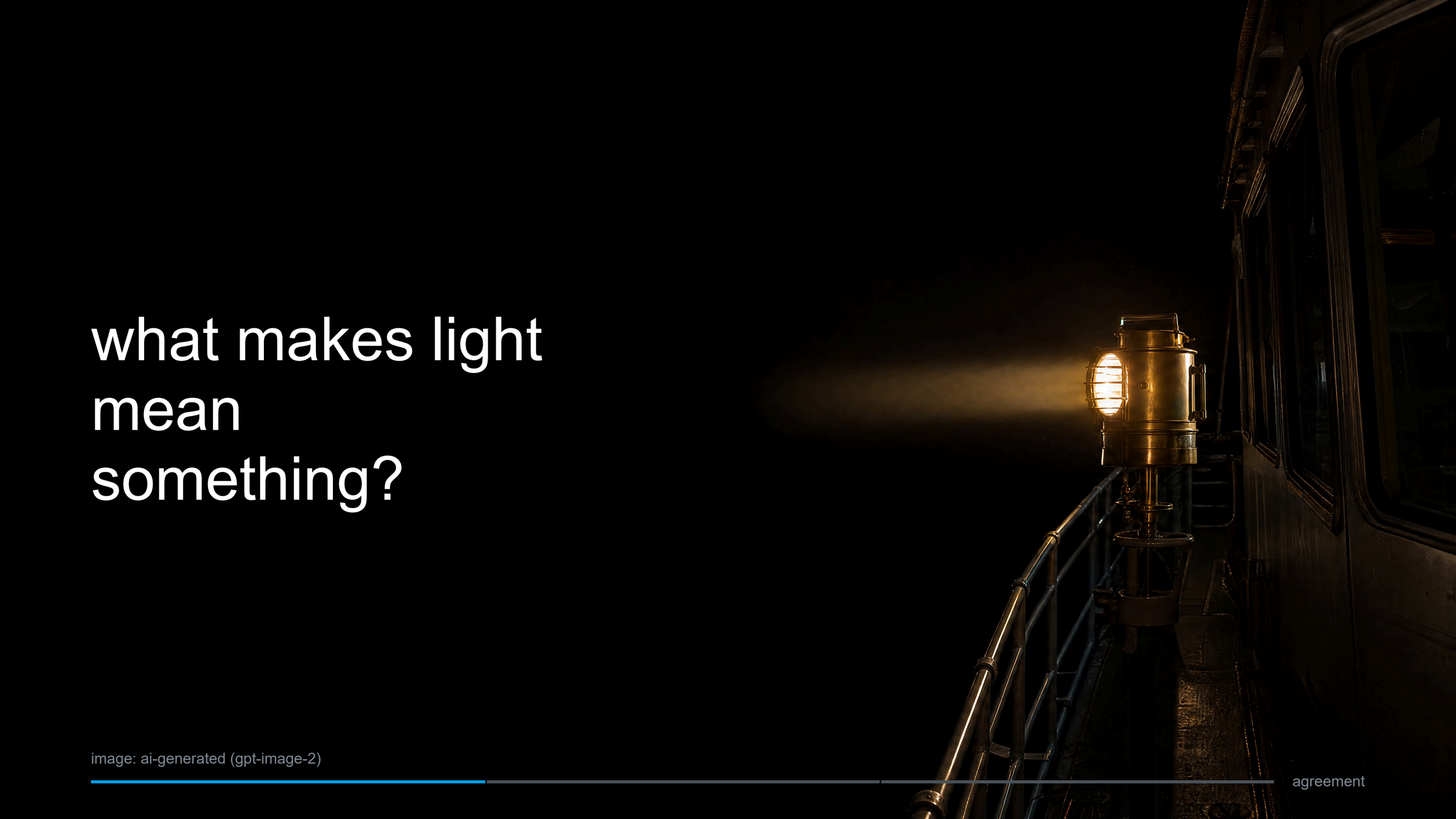


image: ai-generated (gpt-image-2)

a symbol is
an agreed,
distinguishable
state.



image: ai-generated (gpt-image-2.5-flare)

a symbol is an agreed, distinguishable state.

distinguishable: the receiver can tell it apart from the others.



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a symbol is an agreed, distinguishable state.

distinguishable: the receiver can tell it apart from the others.

agreed: both sides gave it the same meaning, beforehand.



image: ai-generated (gpt-image-2.5-flare)

perfect reception, perfect nonsense

the sender's table

red	→	`t`
green	→	`e`
blue	→	`a`
yellow	→	`m`

sent: → `team`

the receiver's table

perfect reception, perfect nonsense

the sender's table

red	→	`t`
green	→	`e`
blue	→	`a`
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sent: → `team`

the receiver's table

red	→	`m`
green	→	`e`
blue	→	`a`
yellow	→	`t`

read as: `meat`

every colour arrived correctly, and still the message did not.

```
ALPHABET = {
    "red":    "00",
    "green":  "01",
    "blue":   "10",
    "yellow": "11",
}
print(ALPHABET["blue"])
```

both sides need this table beforehand, which makes it a promise rather than data.

part 2

counting information

the guessing game, measured

possibilities left

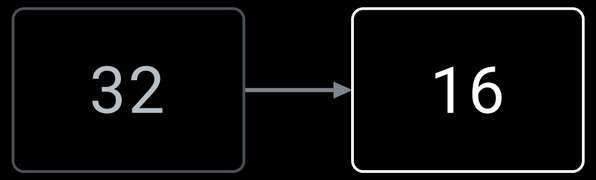
32



32 cards, one of them is wanted

the guessing game, measured

possibilities left

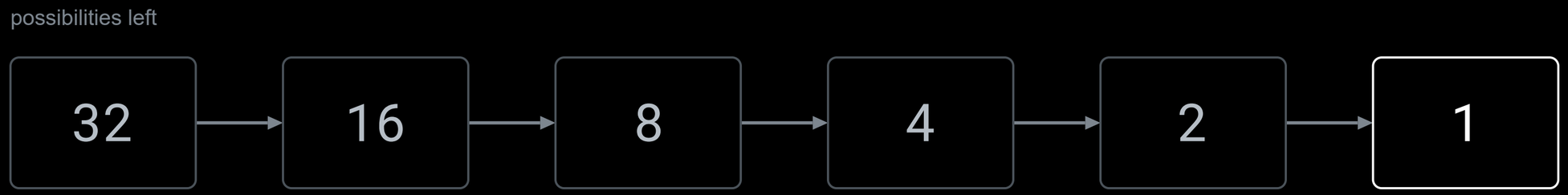




32 cards, one of them is wanted

each yes-or-no question rules out half of what is left

the guessing game, measured





32 cards, one of them is wanted

each yes-or-no question rules out half of what is left

halving finds one card out of 32 in exactly five questions, and not only when you are lucky.

not every answer is worth one bit

`"is it red?"`		
`yes`	→	16 left
no	→	16 left

always exactly one halving

`"is it the ace of spades?"`		
`yes`	→	0 left, solved
no	→	31 left

almost always the second line

you cannot choose which answer you get.

what a question is worth on average

question	answer	chance	you learn	on average
"is it red?"	yes	1/2	1 bit	1 bit
	no	1/2	1 bit	
"is it the ace of spades?"	yes	1/32	5 bit	0.2 bit
	no	31/32	0.05 bit	

a rare answer tells you a lot, while an expected answer tells you little

and in general

in words

how likely an answer is × what that answer is worth, added up

in symbols

$$E = \Sigma\ p_i \cdot \log_2(1/p_i)$$

p_i	how likely that answer is
$\log_2(1/p_i)$	what that answer is worth, in bits

and in general

in words

how likely an answer is × what that answer is worth, added up

in symbols

$$E = \Sigma\ p_i \cdot \log_2(1/p_i)$$

p_i	how likely that answer is
$\log_2(1/p_i)$	what that answer is worth, in bits

the two questions, worked out

$$0.5 \cdot 1 + 0.5 \cdot 1 = 1 \text{ bit}$$

$$0.03 \cdot 5 + 0.97 \cdot 0.05 = 0.2 \text{ bit}$$

a fair split is the only way to get a whole bit.



one bit = one decision between two equally likely possibilities

a light switch holds one bit, and it needs no computer for that.

0

1

one bit = one decision between two equally likely possibilities

$H = \log_2(N)$ uncertainty: how many halvings are still missing

$I = H_1 - H_2$ information: what an answer removed

32 cards mean an uncertainty of 5 bits, and each halving removes 1 bit

a light switch holds one bit, and it needs no computer for that.

what a bigger alphabet buys

colours	bits per symbol
2	$\log_2(2) = 1$
4	$\log_2(4) = 2$
8	$\log_2(8) = 3$
16	$\log_2(16) = 4$

what a bigger alphabet buys

colours	bits per symbol	time for the same file, at the same symbol rate
2	$\log_2(2) = 1$	100 %
4	$\log_2(4) = 2$	50 %
8	$\log_2(8) = 3$	33 %
16	$\log_2(16) = 4$	25 %

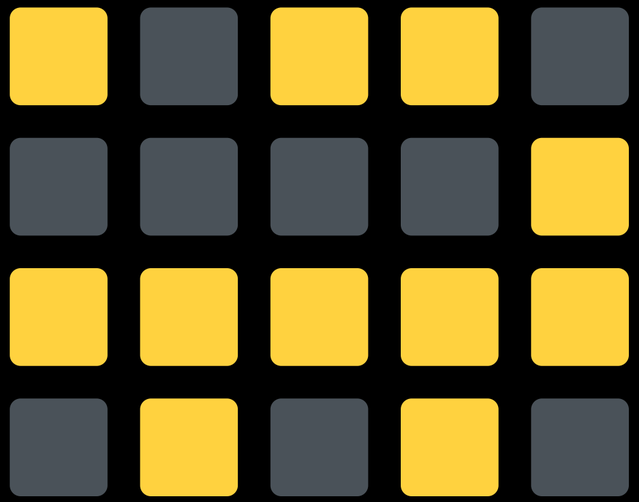
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the file and the symbol rate stay the same, and only the alphabet changes.

two symbols are enough for anything

two symbols, five flashes per character



10110

00001

11111

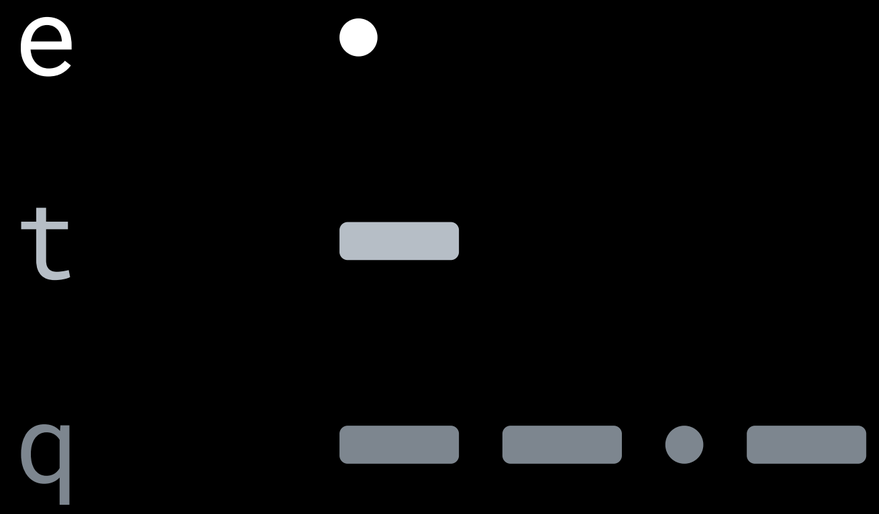
01010

$2^5 = 32$

32 combinations: room for a whole alphabet

what a big alphabet does per symbol, two symbols do in a group.

when symbols are not equally likely



the most common letter

a rare letter

in real text, symbols are not equally likely

rare symbols may cost more, as long as the common ones stay short.

part 3

your alphabet

the payoff

more bits per symbol

the same file needs fewer symbols

and arrives sooner

the price

the colours move closer together

noise reaches across the gap

and symbols are read wrong

the size of your alphabet is not a matter of taste but the result of a measurement.

four steps to your alphabet

agree on your alphabet and write it down
measure a profile for every symbol
teach the receiver to recognise every symbol
send 20 random symbols and count the errors

light means nothing
until you agree what it means.

light means nothing
until you agree what it means.

and how much it means, you can count.